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Intrinsic topological hinge states induced by boundary gauge fields in photonic metamaterials

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Abstract

Higher-order topological insulators (HOTIs) can support boundary states at least two dimensions lower than the bulk, attracting intensive attention from both fundamental science and application sides. Lattice-based tight-binding models such as Benalcazar-Bernevig-Hughes model have driven significant advancements in realizing HOTIs across various physical systems. Here, beyond lattice model, we demonstrate that a cylinder with an arbitrary cross section, composed of a homogeneous electromagnetic medium featuring nontrivial second Chern numbers $c_2 = \pm 1$ in a synthetic five-dimensional space, can exhibit topologically protected HOTI-type hinge states in three-dimensional laboratory space. Interestingly, this hinge state is essentially a chiral zero mode arising from the interaction between Weyl arc surface states, guaranteed by a nontrivial c_2 , and an effective magnetic field induced by the curvature of the cylinder surface. Compared to conventional schemes to generate HOTIs, our approach is more robust, as it is an intrinsic topological phase and therefore does not rely on additional symmetry protections such as time-reversal, parity, or chiral symmetry. We experimentally realize such a cylinder using a photonic metamaterial and confirm the existence of hinge states via microwave near-field measurements. Our work introduces the concept of boundary gauge fields and establishes the link between synthetic-space c_2 and real-space HOTI states, thereby generalizing HOTIs to cornerless systems.

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1 Introduction

The study of topological insulators (TIs) [1, 2] has revolutionized our understanding of condensed matter physics by introducing the concept of topological phase transition. A hallmark of TIs is the presence of robust boundary states at the interfaces between different topological phases, a feature stemming from the principle of “bulk-boundary correspondence”. This principle dictates that a d -dimensional TI will host gapless states on its $(d - 1)$ -dimensional boundaries. For instance, a two-dimensional (2D) TI will exhibit 1D gapless edge states [1–5]. In recent years, a novel class of TIs, termed as HOTIs [6–10], has gained intensive attention. Unlike conventional TIs, HOTIs support gapless states in dimensions lower than $(d - 1)$. In a 2D HOTI, the 1D edge states are either gapped or absent, while the 0D isolated states emerge at the corners [11]. HOTIs have been widely

realized in acoustic systems [12–16], electrical circuits [17–19], waveguide [20, 21], and photonic crystals [22–24], mainly based on two types of lattice Hamiltonians or their modified versions: the decomposable higher-dimensional Su–Schrieffer–Heeger (SSH) model [11, 23] and the Benalcazar–Bernevig–Hughes (BBH) model [6, 7]. Their topological properties are characterized by either multiple quantized dipole moments [11, 23], a fractional charge [25–27], or a quantized higher-order multipole moment [6, 7]. In practice, these tight-binding model (TBM)-based systems are primarily implemented by precise control of coupling strength and phase between adjacent local resonators. Given the intriguing physics and many exciting applications of HOTIs, it is natural to ask whether alternative frameworks beyond coupling-based TBMs can support HOTI behavior.

Important hints can be obtained by examining the response of a natural crystal material, which is determined by the crystal's space-group structure and its chemical composition. The space-group structure dictates how atoms are coupled together, while the chemical composition influences the intrinsic responses of constitutional atoms. In electromagnetic (EM) systems, photonic crystals [28] predominantly focus on controlling lattice symmetry and inter-unit interactions, with an emphasis on space-group structure. In contrast, metamaterials [29], composed of subwavelength microstructures (i.e., meta-atoms) exhibiting tailored optical responses, offer an alternative platform to control EM waves. Under the deep subwavelength condition, metamaterials can be homogenized as effective media exhibiting distinct constitutive relations, enabling arbitrary manipulation of permittivity, permeability, and magneto-electric coupling terms through designing the constitutional meta-atoms [30]. Based on metamaterials with tailored effective-medium properties and thus well-controlled photonic dispersion spectra, a wide range of topological photonic phenomena have been realized [3, 30], including photonic insulators [31, 32], Weyl points (WPs) [33–35], nodal lines [36–38], Dirac points [39, 40], and even Yang monopoles [41, 42] and Weyl surfaces [43–45] in synthetic 5D spaces. The interaction between Yang monopole and gauge fields has been previously investigated, establishing that the 5D chiral zero mode protected by c_2 is physically equivalent to a HOTI-type localized state, but in the Fock space. This suggests an intrinsic connection between these two topological phenomena [46]. Furthermore, it indicates that HOTIs might emerge within specific subspace of a higher dimensional system characterized by a higher-order Chern number, such as the 5D Yang monopole metamaterial (YMM) or Weyl surface metamaterial with a nontrivial second Chern number c_2 [47–49]. In such systems, the intrinsic responses

of artificial meta-atoms may replace the coupling-based mechanisms in SSH and BBH models to offer new possibilities for realizing HOTIs.

In this article, we demonstrate, both theoretically and experimentally, that HOTI-type hinge states can exist in a homogeneous YMM cylinder, as illustrated in Fig. 1. These hinge states are topologically protected by the nontrivial c_2 in a synthetic higher dimensional physical space. The higher-order topological invariant guarantees the existence of nontrivial boundary states, characterized by WPs on arcs connecting the projection of higher dimensional topological structures – such as Yang monopoles or Weyl surfaces with opposite c_2 , known as Weyl arcs [43, 44, 50]. Such synthetic WPs at the boundary will interact with an effective magnetic field induced by the variation of the boundary's normal direction, leading to topologically protected localized chiral zero modes [51–56] that manifest as the HOTI-type hinge states. This mechanism is fundamentally distinct from conventional HOTIs, representing an intrinsic topological phase rather than symmetry-protected topological phases [57], and ensures topological protection across a broad class of electromagnetic materials and for cylinders with arbitrary cross-sections. We construct a metamaterial cylinder exhibiting the desired effective-medium property and confirm the existence of these HOTI-type hinge states through both finite-element method (FEM) simulations [58, 59] and microwave near-field experiments. This work not only provides a new platform for constructing HOTIs but also highlights the significant potential of the effective medium model in exploring novel topological phenomena.

2 Results

2.1 Theoretical Insights: From 5D Metamaterials to 3D HOTI Hinge States

We start from discussing the topological surface properties of a synthetic 5D YMM. This YMM acts as a unique uniaxial medium exhibiting duality symmetry ($\hat{\epsilon} = \alpha \cdot \hat{\mu}$), purely antisymmetric bianisotropy ($\gamma_{iz} = -\gamma_{zi}$), and degenerate electric and magnetic resonances along the z -axis at the plasmonic frequency ω_p . It has been shown in Ref. [43] that such a system behaves like Yang monopoles with nontrivial second Chern numbers $c_2 = \pm 1$ in a synthetic 5D space, constructed by three real momentum dimensions $k_{i=1,2,3}$ and two synthetic dimensions $[k_4, k_5] = \omega \cdot [\gamma_{xz}, \gamma_{yz}] = \omega\gamma_0 \cdot [\cos\psi_0, \sin\psi_0]$. As a key element, ψ_0 denotes the orientation of the bianisotropy vector of the metamaterial. The bulk-boundary correspondence of the topological nontrivial c_2 is manifested by the presence of 3D Fermi hypersurfaces and 1D Weyl arcs at the 4D boundaries of the 5D YMM [43, 44, 50], as shown in Fig. 1(a). In the 3D laboratory subspace, this

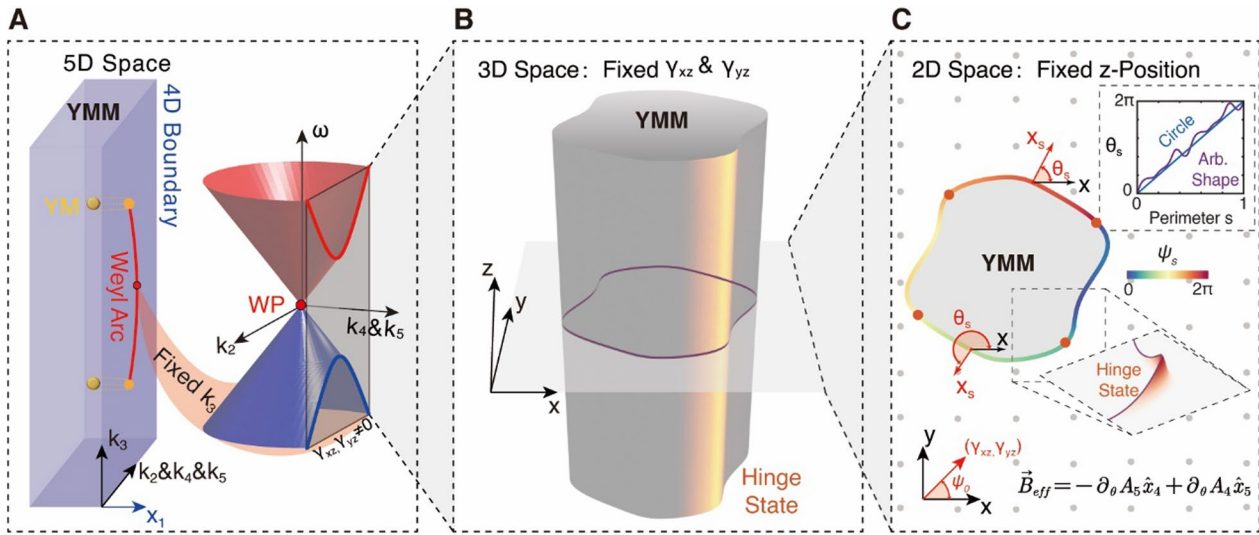


Fig. 1 Illustration of the HOTI-type Hinge States in an arbitrary YMM Cylinder. **a** Illustration of the 1D Weyl arc connecting projections of Yang monopoles with opposite c_2 , on a 4D boundary normal to the x -direction. The right inset depicts the local Weyl dispersion of surface states at a given k_z . **b** Localized HOTI-type hinge states on the side boundary of a 3D cylinder, constructed from a homogeneous electromagnetic medium featuring nontrivial c_2 in a synthetic 5D space. **(c)** Schematic of the intrinsic variation in the effective orientation angle of the bianisotropy vector $\psi_s = \psi_0 - \theta(s)$ manifested in the surface Hamiltonian, along an arbitrary $x - y$ cross-section. The top-right inset illustrates the consistent 2π variation of the normal vector orientation $\theta(s)$ along the perimeter, irrespective of the special shape of the cross section. The bottom-right inset displays the localized energy density field distribution of a representative hinge state, with orange dots indicating the localization positions of all four hinge states along the perimeter at $k_C = 0$

topological invariant guarantees the existence of synthetic WPs at the interface between the YMM and air when $|k_z| < k_{YM}$. On the surface normal to the x -direction, the surface Hamiltonian at a given k_z can be analytically derived using the $k \cdot p$ approximation method:

$$H_{WP} = \omega_C \sigma_0 + v_1 k_y \sigma_1 + v_2 k_4 \sigma_2 + v_3 (k_5 - k_C) \sigma_3 \quad (1)$$

where $\omega_C = \omega_{0C} + v_0 \cdot (k_5 - k_C)$. The space of Pauli matrices σ_i is spanned by degenerate TE- and TM-polarized surface states at the WP location. The k_z -dependent group velocity v_i and the WP degeneracy point $[k_C; \omega_{0C}]$ are determined by the specific parameters of the YMM, with $k_C = 0$ when the YMM and air satisfy the impedance matching condition at $\alpha = 1$. This Weyl formulation highlights the intrinsic topological properties of surface states inherited from the higher dimensional Yang monopoles. Notably, k_4 and k_5 , defined by bianisotropy, act as fixed mass terms in the 3D laboratory space, generally resulting in gapped rather than gapless surface states.

This surface WP can interact with an effective magnetic field induced by the curvature of the surface, unveiling a novel mechanism to generate HOTI-type hinge states – a concept proposed here for the first time. Consider a cylinder oriented along the z -axis and constructed

from such a homogenous YMM, as shown in Fig. 1(b). In any arbitrary cross-section within the $x - y$ plane, the boundary profile can be defined by the direction of the local normal vector $\vec{x}_s$ along the perimeter, denoted as $\theta(s)$ in a fixed coordinate system, as shown in Fig. 1(c). Here, s represents the position along the perimeter, ranging from 0 to 1. As s traverses the entire perimeter, $\theta(s)$ inevitably undergoes a full evolution of 2π . Being inherently bianisotropic material parameters, the relative orientation angle of the bianisotropy vector $\psi_s = \psi_0 - \theta(s)$ along the perimeter depends on the choice of the local coordinate frame. Because $\theta(s)$ unavoidably varies along the side boundary, the surface Hamiltonian H_{WP} undergoes a local coordinate transformation, even though the material parameters of the cylinder system are homogeneous. Since the c_2 -protected surface WP remains robust against changes in boundary orientation, this curvature-induced transformation simply shifts the local position of the surface WP along varying $\theta(s)$, effectively generating an artificial gauge potential. This potential couples the surface WP with an equivalent magnetic field, leading to the emergence of a localized chiral zero mode manifested as a hinge mode.

Assuming a sufficiently large cross-section to neglect finite-size effects, the global surface Hamiltonian parameterized by θ along the perimeter is expressed as:

$$H_{GS} = \omega_C \sigma_0 - v_1 \cdot i k_R \partial_\theta \cdot \sigma_1 + \left\{ \begin{array}{l} v_2 \cdot \omega_{0C} \gamma_0 \cos(\psi_0 - \theta) \cdot \sigma_2 \\ + v_3 \cdot [\omega_{0C} \gamma_0 \sin(\psi_0 - \theta) - k_C] \cdot \sigma_3 \end{array} \right\} \quad (2)$$

where $k_R = 1/R_\theta$ is determined by the local radius of curvature R_θ along the perimeter. This global surface Hamiltonian can be directly solved using a 1D plane wave expansion method, due to the effective periodicity $H_{GS}(\theta) = H_{GS}(\theta + 2\pi)$. Such a semi-analytical approach effectively characterizes the dispersion spectra and field distributions in the YMM cylinder, aligning well with the numerical solutions obtained from FEM simulations. Upon careful examination of the localized surface modes near the surface bandgap, four distinct sets of HOTI-type hinge modes are clearly identified, as illustrated in

Fig. 2(a-d) for a circular YMM cylinder with $\alpha = 1$. These hinge states are located near $\theta_{c,n} = \psi_0 - n \cdot \pi/2$, corresponding to $\psi_S = n \cdot \pi/2$, where $n = 0, 1, 2, 3$. Notably, in this circular cylinder, the structure lacks anisotropy, indicating that the hinge states originate solely from the intrinsic properties of the electromagnetic medium.

To gain deeper insights into the origin of the discovered HOTI phenomenon, we further analyze the behavior in the vicinity of the localized hinge states, as shown in Fig. 2(e-f). Taking a special localized position $\theta_c = \psi_0 - \pi/2$ (i.e., $\psi_S = \pi/2$) as an example, the local surface Hamiltonian can be approximatively expressed as:

$$H_{LS} = \omega_C \sigma_0 - v_1 \cdot i k_R \partial_\theta \cdot \sigma_1 + v_2 \cdot \omega_{0C} \gamma_0 \delta \theta \cdot \sigma_2 + v_3 \cdot (\omega_{0C} \gamma_0 - k_C) \cdot \sigma_3 \quad (3)$$

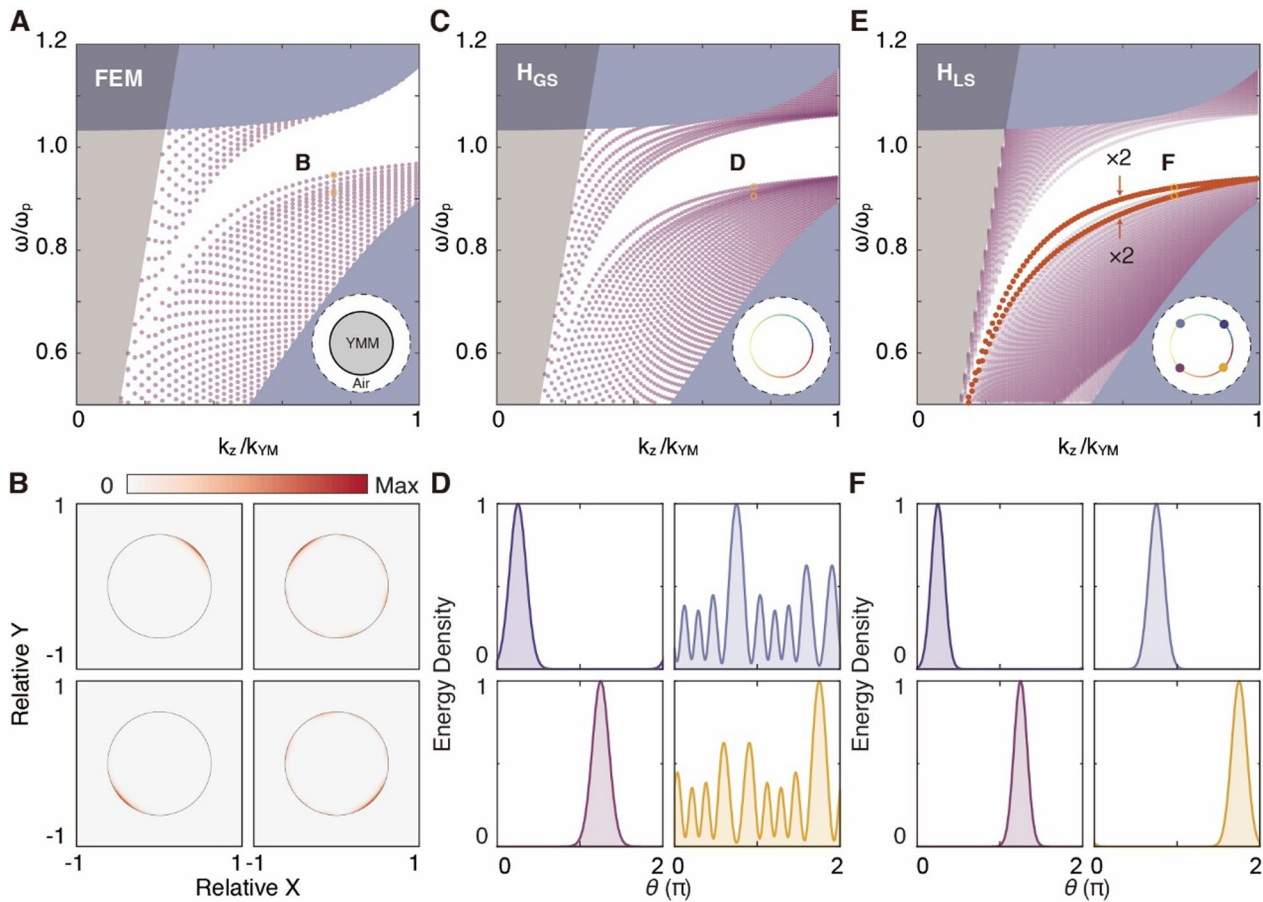


Fig. 2 Localized Hinge States in the YMM Cylinder under Different Models. **a** Dispersion spectra for a circular YMM cylinder calculated using FEMs, with $\alpha = 1$ and $\psi_0 = \pi/4$. **b** Energy density distributions for four HOTI-type hinge-localized states, marked by circles in panel (a). **c-f** Similar to **a, b** but calculated using: **c, d** a 1D plane wave expansion of the varying global surface Weyl Hamiltonian H_{GS} , and **e, f** four local Weyl Hamiltonians H_{LS} coupled with an effective magnetic field. The orange lines in **e** indicate the chiral zero modes, while the purple and gray regions represent the bulk band region of the YMM and air, respectively

with $\delta\theta = \psi_0 - \theta \ll 1$. This surface Hamiltonian can be mapped to a standard form: $H = \sum_{i=1}^3 v_i \cdot [\tilde{k}_i - \tilde{A}_i(\vec{x})] \cdot \sigma_i$, representing a WP coupled with an external magnetic field. Here, the effective gauge field $\tilde{A}_2 = -\omega_{0C}\gamma_0 \cdot \delta\theta$ arises from the variation of the boundary's normal direction $\theta(s)$, functioning as a magnetic field $\vec{B}_{\text{eff}}$ in the synthetic $x_4 - x_5$ plane and coupling with the synthetic surface WP. This magnetic field topologically induces a chiral zero mode in the synthetic surface Weyl semimetal [51–56], which emerges as a localized Hermite-Gaussian mode near θ_c and manifests as a HOTI-type hinge state on the cylinder's side boundary. Localization of this hinge state depends sensitively on the strength of bianisotropy γ_0 and the local curvature radius R_θ . As R_θ increases to infinity, corresponding to a flat surface, or as γ_0 decrease to zero, corresponding to an isotropic medium, this mode evolves from being confined to spreading out as a surface mode. Conversely, as R_θ approaches zero, where the cross-section locally forms a sharp corner connecting two distinct uniform surface Hamiltonians, this localized mode transitions to a topologically equivalent edge state, resulting in a more localized HOTI-type hinge state on the cylinder with a sharp corner. Furthermore, the frequency of this hinge state is determined by $\tilde{k}_3 = \omega_{0C}\gamma_0 - k_C$, which depends on the impedance of the YMM and is not necessarily at the effective zero frequency $\omega = \omega_{0C}$.

A series of such topologically special angles $\theta_{c,n}$ exist on the cylinder, satisfying $\cos(\psi_0 - \theta_C) = 0$ or $\sin(\psi_0 - \theta_C) = k_C/\omega_{0C}\gamma_0$, and hosting localized chiral zero modes through a similar mapping mechanism, which is in perfect agreement with the numerical results for $k_C = 0$. Away from these specific angles, the spatially varying bianisotropy terms locally generate non-zero constant terms in the leading order, shifting the localization to a position beyond the scope of the local approximation, thereby suppressing the existence of localized chiral zero modes. Consequently, for any arbitrary cylindrical configuration described by $\theta(s)$, there are always a total of $4 + 2m$ (or $2 + 2m$ when $k_C > \omega_{0C}\gamma_0$) localized hinge states near the points $\theta(s) = \theta_{c,n}$, where m is a non-negative integer used to mark the additional intersection points between $\theta(s)$ and $\theta_{c,n}$.

Due to discrepancies between the local approximation and the slowly varying H_{GS} Hamiltonian, we note that H_{LS} may exhibit a slight deviation in predicting the chiral zero mode. However, this deviation does not affect the topological nature of these hinge modes. The local model H_{LS} indicates that the hinge states are topologically protected by the second Chern number c_2 , in the region where Weyl arcs exist, stemming intrinsically from the variation of surface Weyl Hamiltonian along the

cylinder's side boundary. Therefore, these hinge modes are an intrinsic topological phase rather than a symmetry-protected one [57], which does not rely on additional symmetry protection, such as $\mathcal{TP}$ symmetry [60–62] ($\mathcal{T}$ is time-reversal symmetry, and $\mathcal{P}$ is parity symmetry), mirror symmetry [9], rotation symmetry [9, 10] or chiral symmetry [6, 49]. Instead, their existence is solely ensured by the robustness of c_2 , distinguishing them fundamentally from the conventional tight-binding systems. Consequently, the emergence of these hinge states is independent of both the system's geometric configuration and the specific effective medium, encompassing not only the 5D YMM but also Weyl surface metamaterial associated with a nontrivial c_2 . Numerical validation across various geometries and media, as illustrated in Fig. S9, highlights the robustness and universality of these topological hinge states.

2.2 Experimental investigation of 3D HOTI hinge states

We design a realistic YMM based on metallic helical structures, as illustrated in Fig. 3, and employ FEM simulations to analyze its photonic band structure. The sample is constructed by stacking printed circuit board (PCB) layers, with each unit cell containing four columns of helix pairs arranged to maintain mirror symmetry. The orientation of each column is carefully designed to break the C_4 symmetry and introduce nonzero anti-symmetric bianisotropic terms, satisfying $\gamma_{iz} = -\gamma_{zi}$ and $\gamma_{xz} = \gamma_{yz}$, as shown in Fig. 3(a), corresponding to an in-plane bianisotropy vector aligned at a 45° angle to the x -axis. This sample can be modeled as the aforementioned effective medium using an RLC-based circuit model [46], which closely matches the predicted dispersion spectra in simulation, as illustrated in Fig. 3(b). The nonzero anti-symmetric bianisotropic terms, functioning as synthetic dimensions k_4 and k_5 , act as effective masses in the 3D laboratory subspace. These terms lift the four-fold degeneracy, opening a band gap and leading to two pairs of degenerate bands.

We next construct a finite-sized YMM cylinder with a square-shaped cross-section, and numerically demonstrate the existence of HOTI-type hinge states on its side boundary, as shown in Fig. 3(c, d). By carefully selecting the permittivity of the PCB substrate $\epsilon_{PCB} = 8.15$ and adjusting the impedance mismatch between YMM and air, one topologically protected hinge states, indicated by orange circles, is positioned exactly within the bandgap of the gapped surface states, represented by purple points, as shown in Fig. 3(c). Three additional topological hinge states overlap with the surface state spectra, forming localized modes akin to bound states in the continuum (BICs) or quasi-BICs. All these localized states reside within the bandgap of the gapped bulk states, shown by

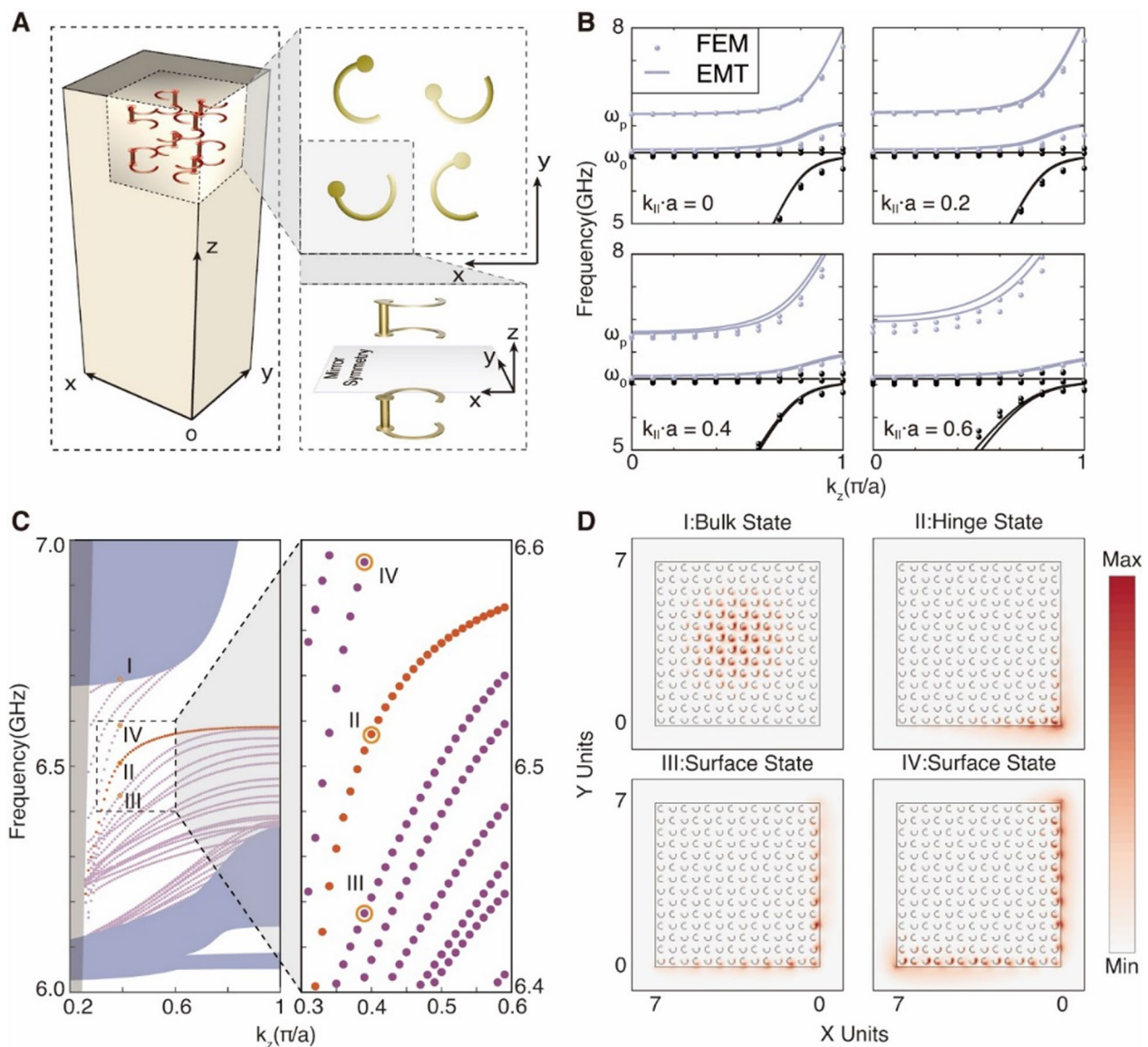


Fig. 3 Schematic diagram of the designed YMM used to realize HOTI. **a** Schematic of the YMM cylinder, with one unit cell highlighted and zoomed in. The inset shows the top view and side view of the metallic helix unit cell. **b** Bulk dispersion of the periodical YMM, where dots represent the FEM simulated results and lines correspond to predictions from RLC-based EMT model. **c** Dispersion spectra along the axial direction k_z for the square metamaterial cylinder, comprising 7×7 unit cells in the $x - y$ cross-section, with the isolated hinge state within the surface bandgap highlighted in orange. **d** $|E|$ field distributions corresponding to four representative eigenstates, highlighted by circles in panel **c**

the purple regions. Figure 3(d) illustrates the representative $|E|$ field distributions in the $x - y$ cross-section for different types of states. Besides the conventional bulk modes spread throughout the entire metamaterial region and the surface states concentrated along the boundary, there is a distinct state entirely localized at the lower right corner of the cross-section, which manifests as the HOTI-type hinge state.

We fabricate a sample according to the design and employ near-field scanning technique to experimentally

characterize these distinct states. The experimental sample is constructed by stacking 480 PCB layers, corresponding to 80-unit cells along the z -direction, with a 7×7 unit cross-section, as shown in Fig. 4(a). The excited wave is launched by a vertically positioned monopole antenna, placed either at the center or at the lower right corner of the bottom edge to selectively target surface and hinge states. Field distributions are measured by the near-field scanning using a monopole or dipole antenna, providing spatially dependent dispersion spectra along k_z

via Fourier transformation. This configuration provides sufficient spatial and wavevector resolution to distinguish hinge modes from surface modes.

The measured dispersion spectra, shown in Fig. 4(b, c), exhibiting distinct dispersion results for surface and hinge states, agreeing well with the predictions from both effective medium theory (EMT) and the full-wave simulations. The surface states occupy most of the k -space within the bandgap of the bulk states, while the hinge state is confined to specific k -positions. Furthermore, there is a clear bandgap for the surface states, with the hinge state positioned exactly within this bandgap, as shown in Fig. 4(d). Field distributions at representative frequencies are presented in Fig. 4(e, f), illustrating the surface states on the $x - z$ and $y - z$ planes, as well as the

localized hinge state along the z -axis. Both types of states are confined to the YMM sample and decay rapidly in air. The surface states extend across the entire side boundary, exhibiting plane-wave characteristics, while the isolated hinge state is strongly localized along the sharp corner of the square cylinder sample, clearly confirming its HOTI property. Such difference in field distributions remains consistent across measurements at different frequencies. Similarly, the dispersion spectra for other three hinge modes, despite being embedded inside the surface state spectra, have also been clearly validated experimentally. These spatially resolved experimental results provide strong evidence for the existence of the HOTI-type hinge state in the YMM cylinder.

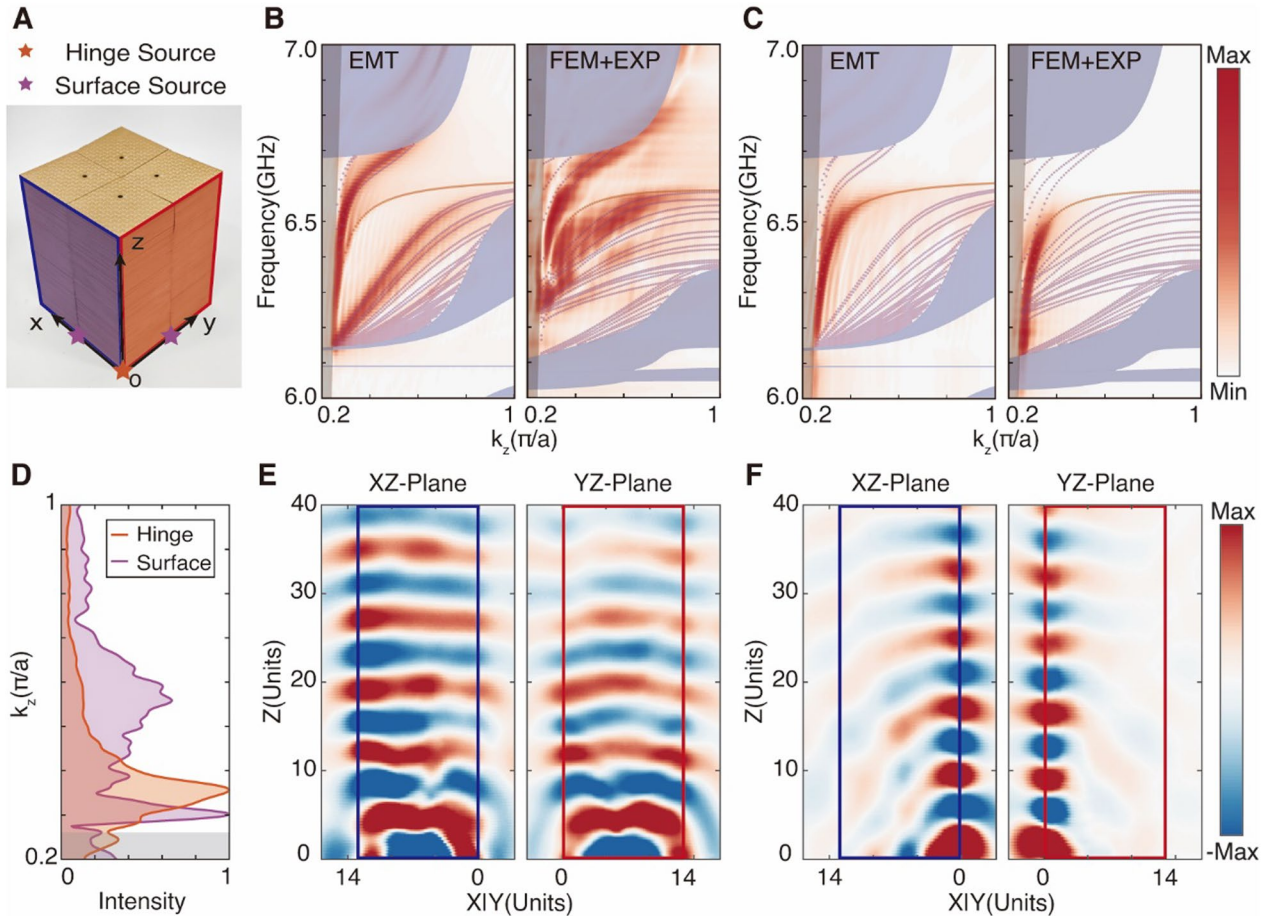


Fig. 4 Experimental observations of the surface and hinge states in the square YMM cylinder sample. **a** Photograph of the YMM sample, fabricated using printed circuit board technology, with side boundaries highlighted by the red and blue boxes. **b, c** Simulated (left) and measured (right) dispersion spectra for: **b** surface and **c** hinge states. The color maps represent the Fourier-transformed field distribution from (left) the simulated EMT model and (right) experimental measurements. **d** The k_z -dependent measurement strength of the surface and hinge states at a representative frequency $f = 6.50$ GHz, with the hinge state located exactly within the bandgap of the surface states. **e, f** The excited field distributions at representative frequencies, measured on two side boundaries of the square YMM cylinder sample, showing results for **(e)** surface excitation at $f = 6.35$ GHz, and **(f)** hinge excitation at $f = 6.22$ GHz

3 Discussions

In summary, we have realized a novel class of HOTIs by utilizing the intrinsic properties of meta-atoms in a photonic metamaterial platform, and experimentally demonstrated the existence of hinge states in a homogeneous YMM cylinder with a square-shaped cross-section. The nontrivial c_2 guarantees that synthetic WPs naturally arise on the surfaces of the YMM, while the unavoidable curvature of the cylinder's side boundary acts as an effective magnetic field leading to the emergence of topological chiral zero modes that manifests as hinge states. This HOTI mechanism fundamentally differs from conventional HOTIs, as it represents an intrinsic topological phase rather than a symmetry-protected one. In particular, it cannot be explained by quantized dipole or multipole and differs from the $\mathcal{TP}$ -invariant real Chern insulator. Such a framework provides a new perspective for realizing HOTIs without relying on crystalline symmetries or discrete lattice structures. Our work reaffirms the power and versatility of utilizing constitutive relation in metamaterials to explore and harness novel topological effects. The proposed platform offers a promising route toward experimental realization of topological states in broader classes of materials, geometries and frequency regimes.

4 Methods

4.1 Numerical simulation setup

A FEM simulation is performed to analyze the eigenstates of the YMM cylinder, based on the RLC-based EMT model with additional auxiliary variables introduced to incorporate resonance effects. The simulation is implemented in Comsol Multiphysics using a manually defined weak form:

$$I_{E\&q} = \int d\tau \cdot \{ \mathcal{D}\tilde{E}^\dagger \cdot \mathcal{D}\tilde{E} - \varepsilon_{bg}\omega^2 \cdot \mathcal{G}\tilde{E}^\dagger \cdot \mathcal{G}\tilde{E} - \tilde{\mathbf{x}}^\dagger(\omega^2 - \omega_0^2 + i\omega\Gamma)\tilde{\mathbf{x}} + \varepsilon_{bg}^{-1}\tilde{\mathbf{x}}^\dagger f_E f_E^\dagger \tilde{\mathbf{x}} \} \quad (4)$$

$$\mathcal{D}\tilde{E} = c\nabla \times \tilde{E} - i\omega g_H^\dagger \tilde{\mathbf{x}} \& \mathcal{G}\tilde{E} = \tilde{E} + i\frac{1}{\varepsilon_{bg}\omega} f_E^\dagger \tilde{\mathbf{x}}$$

Here, $\tilde{E}^\dagger$ and $\tilde{\mathbf{x}}^\dagger$ are the test functions, which are, by default, selected as the basis functions used to interpolate the electric and auxiliary field. The vectors f_E and g_H specifically represent the dipole responses of each helical resonance. This weak form and its related formulations are applied to analyze the dispersion spectra, eigenstate field distributions, and excitation response behavior of the YMM cylinder.

4.2 Experimental setup

The sample consists of 480 stacked PCB layers, forming 80-unit cells along the z direction, with a 7×7

unit cross-section. Four identical samples are assembled into different configurations to either extend the length along the z-direction for improved k_z resolution or enlarge the side measurement area to capture field localization on the square cylinder boundaries. Field amplitude and phase are collected using a vector network analyzer (VNA, CETC-3671D, China Electronics Technology Group Co., Ltd), connected to two electric antennas serving as the source and the probe. The probe antenna is mounted on a high-precision displacement stage, with a motion error of less than 0.02 mm, allowing high-resolution near-field scanning. A dipole antenna (XB-ODA6-8S, Beijing Xibao Electronic Technology Co., Ltd), with a gain of 2 dB in the 6–8 GHz range, is used to resolve polarization-selective surface responses.

Supplementary Information

The online version contains supplementary material available at <https://doi.org/10.1186/s43593-025-00097-7>.

Additional file 1.

Author contributions

S.M. conceived the initial concept, C.H. and L.Z. participated in the experimental measurement, C.H., L.Z. and S.M. performed numerical simulations and analytical calculations, and S.M., L.Z. and S.Z. supervised the project.

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Availability of data and materials

The data that support the findings of this study are openly available from Science Data Bank at <https://doi.org/10.57760/sciencedb.20619>.

Declarations

Competing interests

The authors declare no competing interest.

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